Optimum Component Values for a Lossy Class E Power Amplifier

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***Abstract* — The Class E power amplifier offers a means to increase the battery efficiency in wireless terminals. The losses can be taken into account by solving the drain waveforms for a switch having a non-zero on resistance. In this paper a new analysis formulation is presented in which the input values (P_{OUT}, V_{DD} and R_{ON}) are combined into one parameter k. Then, the efficiency and optimum component values for a lossy Class E amplifier can be solved with relatively simple calculations. The analysis reveals that there is a maximum value for switch losses that can be accepted in order to operate in the Class E mode. The results of the analysis are presented in plots providing initial component values for practical Class E power amplifier design. The validation simulation results show excellent match with the calculated values.**

I. INTRODUCTION

Battery efficiency improvement is a key research topic in wireless terminals. While the power consumption of the digital section reduces along with the down-scaling of the CMOS technology, the radio frequency section continues to dominate the power consumption. In particular, the transmitter efficiency is a key parameter in this respect. High efficiency improves the operation time and relaxes the heat transfer requirements.

For the highest possible transmitter efficiency in wireless terminal, switch-mode power amplifiers have been considered. In particular Class E amplifier, invented already in the early 1970's by Sokal and Sokal [1], seems to be suitable for radio frequency operation. Several implementations have recently been reported, e.g., [2], [3]. Since the Class E power amplifier concept is really defined through its waveforms, it can be questioned whether these implementations having efficiencies below 70% really operate in the Class E mode.

The heart of the Class E power amplifier is the switch. In traditional Class E analysis the switch is assumed to be ideal; consequently, either the voltage over the switch or the current through the switch is zero. In real power amplifiers the switch is typically implemented using a FET that always has some resistive losses involved with it.

Some previous studies have addressed the switch losses in a Class E amplifier. Raab and Sokal presented already in 1978 a simple approximate method for calculating the effect of power losses in the switch [4]. More recently, Kessler et al. published a method for analyzing the effect of resistive losses in each of the Class E amplifier components [5]. While a variable duty cycle was considered, the waveforms of the amplifier were not reformulated. Parallel to the work in this paper, Wang et al. developed an improved design methodology for Class E amplifier design with finite switch on resistance [6]. They had a very similar starting point to this work: first consider the design specifications and then solve the resulting component values. The procedure, however, ends up being quite different, because they use a fixed $R_{ON}C_{DS}$ product, whereas in this work the shunt capacitor is one of the design parameters. This is beneficial especially in GaAs Class E power amplifiers, where transistor C_{DS} is not necessary sufficiently large for the Class E shunt capacitor.

The objective of this paper is to analyze the effect of the finite on resistance in the switch of the Class E amplifier with a simple reformulation of the drain waveforms. With the new waveforms the respective optimum component values are solved. As a result, also a new condition for maximum switch losses for Class E operation is derived.

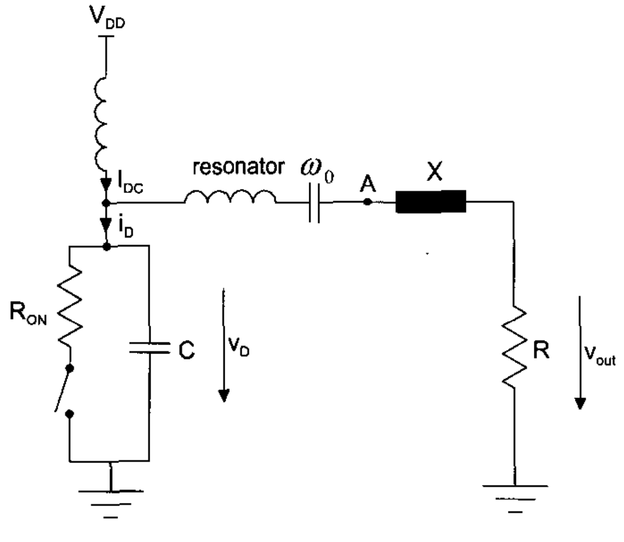

Fig. 1. The Equivalent circuit for the lossy Class E amplifier.

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II. Assumptions

The analysis is based on the following assumptions:

- Switch off-resistance is high: there is no current through the switch during the off-state.
- Voltage over the switch during the on-state is caused only by the resistive channel losses.
- Other loss mechanisms involved (dc feed inductor losses, matching circuit losses, losses involved in the finite transition time etc.) need to be considered separately. E.g., passive matching circuit losses can be absorbed into the load resistor in the analysis.
- Switch duty cycle is 50%.
- Resonator Q-value is high enough to suppress the harmonics

Accordingly, the amplifier can be represented with the basic equivalent circuit shown in Fig. 1. In the optimum circuit the drain voltage of the open switch decreases smoothly to zero at the switch closing point. Consequently, the operation is very similar to the basic lossless Class E amplifier. With the smooth transition no energy is stored in the capacitor when the switch is closed; therefore, the switch power dissipation can be assumed to be dominated by the transistor losses. The resulting shapes of the drain voltage and current waveforms are shown in Fig. 2.

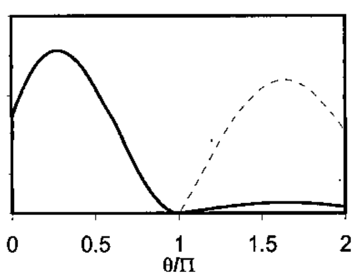

Fig.2. Drain voltage (solid line) and current waveforms (dotted line) of the lossy Class E amplifier.

III. Analysis

Based on the assumptions the lossy Class E definition is:

$$\begin{aligned} & i_D(\theta) = 0 \text{, when } 0 < \theta \le \pi \\ & v_D(\pi) = 0 \text{ and} \\ & v_D(\theta) = i_D(\theta) R_{ON} \text{, when } \pi < \theta \le 2\pi \end{aligned} \tag{1}$$

We start the analysis by calculating the waveforms. The switch current is:

$$i_D = I_{DC} - \frac{v_{out}}{R}\sin(\theta + \phi) \tag{2}$$

When the switch is open this current is charging the shunt capacitor. Accordingly, we can solve the voltage waveform by integrating the current:

$$\frac{1}{\omega C}\left\{ I_{DC}\theta + \frac{v_{out}}{R}\left[\cos(\theta+\phi) - \cos\phi\right]\right\} = v_D - v_1 \tag{3}$$

Voltage v_I is the startup voltage, which is determined by the on-state current

$$v_1 = R_{ON} i_D(2\pi) = R_{ON}\left(I_{DC} - \frac{v_{out}}{R}\sin\phi \right) \tag{4}$$

Substituting (4) into (3) yields the drain voltage waveform. The voltage is guaranteed to be continuous at $\theta = 0$, because of the selection of the boundary value for the integration.

Load phase angle ϕ can be solved by applying the Class E requirements $i_D(\pi)=0$ and $v_D(\pi)=0$. Simultaneously, these conditions guarantee the continuity at $\theta = \pi$. The current condition becomes:

$$i_D(\pi) = I_{DC} - \frac{v_{out}}{R}\sin(\pi + \phi) = 0 \Rightarrow \sin\phi = -\frac{I_{DC}R}{v_{out}} \tag{5}$$

So this condition is exactly the same as in the basic Class E theory. For the voltage condition, a new parameter y is introduced:

$$y = \omega C R_{ON} \tag{6}$$

The voltage condition becomes after substituting $\sin(\phi)$ into the voltage waveform

$$\cos\phi = \frac{I_{DC}R}{v_{out}} \frac{(2y+\pi)}{2} \tag{7}$$

Now we can solve $\tan(\phi)$

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{-2}{(2y+\pi)} \tag{8}$$

The final waveforms can be derived by substituting $\sin(\phi)$ and $\cos(\phi)$ into (2) and (3)

$$v_D = \frac{I_{DC}}{\omega C}\left\{ \theta + \left(\frac{\pi}{2} + y\right)\cos\theta + \sin\theta - \frac{\pi}{2} + y \right\} \tag{9}$$

when $0 < \theta \le \pi$. The current for $\pi < \theta \le 2\pi$ becomes

$$i_D = I_{DC}\left[1 - \left(\frac{\pi}{2} + y\right)\sin\theta + \cos\theta \right] \tag{10}$$

The next step is to calculate the average drain voltage that needs to be equal to the drain supply voltage. Integration yields

$$V_{DD} = \frac{I_{DC}}{2\pi\omega C}\left(2 + 3\pi y + 2y^2\right) \tag{11}$$

To be able to solve the efficiency, switch losses are calculated

$$P_{LOSS} = \frac{1}{2\pi}\int_0^{2\pi} i_d(\theta)v_d(\theta)d\theta \tag{12}$$

The losses are only present when the switch is conducting; thus, the integration yields

$$P_{LOSS} = R_{ON}I_{DC}^2\left(\frac{7}{4} + \frac{\pi^2}{16} + \frac{\pi}{4}y + \frac{2}{\pi}y + \frac{1}{4}y^2\right) \tag{13}$$

Now, a new parameter k is introduced

$$k = \frac{P_{OUT}R_{ON}}{V_{DD}^2} \tag{14}$$

The output power of the amplifier is

$$P_{OUT} = P_{DC} - P_{LOSS} \tag{15}$$

A solution can be found by substituting (11), (13) and (14) into (15). As a result, we get a forth-order equation for y:

$$y^4\left(k + \frac{\pi^2}{4}\right) + y^3\left(3\pi k + \pi + \frac{\pi^3}{4}\right) + y^2\left(2k + \frac{9\pi^2}{4}k + \frac{\pi^2}{4} + \frac{\pi^4}{16}\right) + y(3\pi k - \pi) + k = 0 \tag{16}$$

The procedure is such that the design specifications (P_{OUT}, V_{DD}) and the device characteristics (R_{ON}) are combined into one input parameter k. Equation 16 is then numerically solved for that particular k resulting in an optimum y value. It is interesting to notice that the equation does not have positive roots above $k \approx 0.100152$. Thus, we can make a conclusion that the requirement to be able to design a Class E amplifier with a lossy switch is

$$R_{ON} < 0.100152\frac{V_{DD}^2}{P_{OUT}} \tag{17}$$

Now with the solved y values we can calculate the efficiency η

$$\eta = \frac{P_{OUT}}{P_{DC}} = \frac{V_{DD}^2 k}{R_{ON}I_{DC}V_{DD}} = \frac{k}{\pi y}\left(1 + \frac{3\pi}{2}y + y^2\right) \tag{18}$$

The results of the amplifier efficiency are presented in Fig. 3.

The optimum capacitance values can directly be solved from the y values.

$$C = \frac{y}{R_{ON}\omega} = \frac{yP_{OUT}}{k\omega V_{DD}^2} \tag{19}$$

The results can be normalized to the ideal lossless Class E shunt capacitor

$$C' = \frac{C}{C_{lossless}} = \frac{\pi y}{k} \tag{20}$$

The calculated values are show in Fig 4.

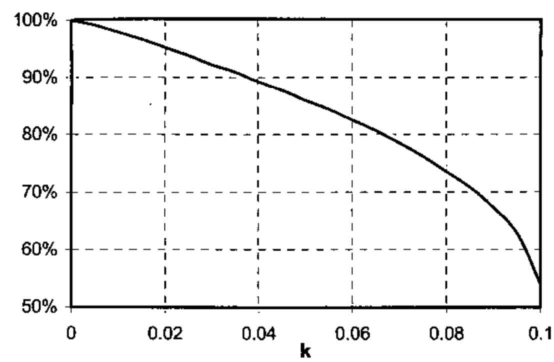

Fig.3. Efficiency of the lossy Class E amplifier.

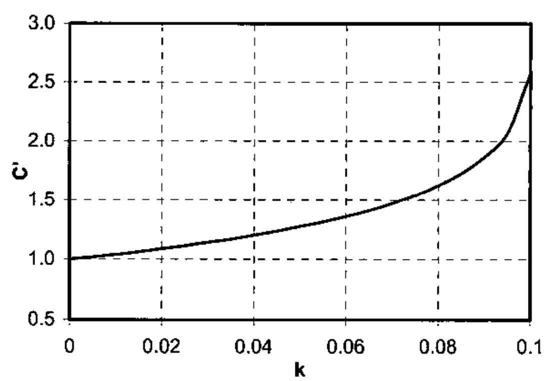

Fig. 4. Resulting optimum shunt capacitor values normalized to the capacitor value in the lossless case.

Now we can solve the optimum values for the remaining components. The resonator capacitance and inductance values can be directly defined based on the intended operating frequency and Q-value. Then, the drain voltage waveform is Fourier-transformed in order to solve the fundamental frequency phase angle ϕ_l and amplitude a_l of the signal at Node A shown in Fig. 1. The solution follows the basic Class E analysis with the new voltage waveforms. To achieve the correct phase at the load an excess reactance X is added in series with the load resistance. The values for R and X are calculated using the solved a_1 and ϕ_1

$$R = \frac{v_{out}^2}{2P_{out}} = \frac{a_1^2}{2P_{out}}\left(1 + \left(\frac{\tan\phi_1 - \tan\phi}{1 - \tan\phi_1 \tan\phi}\right)^2\right)^{-1} \tag{21}$$

$$X = R\tan(\phi_1 - \phi) \tag{22}$$

The equations for the R and X are relatively complex rational equations that can be solved in a straightforward manner. The resulting values for R and X are presented in Figure 5. The results are normalized to the ideal lossless Class E shunt component values. Now all the required components–C, R, and X–are solved as a function of the input parameter k.

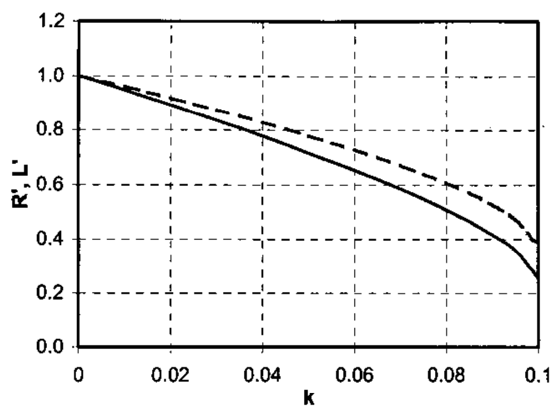

Fig. 5. Resulting optimum load resistor (solid line) and excess inductance (dotted line) values normalized to the values in the lossless case.

IV. Method Validation

The method was validated with time-domain circuit simulations. The test circuit was an ideal Class E amplifier with an ideal switch having resistance at the on-state. The target specifications for the amplifier were: P_{OUT} = 1.0 W, V_{DD} = 2.0 V, and f_0 = 900 MHz. The performance of the circuit was simulated using different values for the switch on-state resistance. The component values were adjusted based on the method presented in this paper. The verification results are shown in Table I. The results prove that the method was capable of providing correct component values for the wanted power level. Furthermore, the method was able to predict the efficiency with excellent accuracy. The simulations also qualitatively prove that k = 0.10 is the maximum allowable value in order to maintain the Class E waveforms.

TABLE I
Method Validation Results

R_{ON} Ω	k	C_D PF	R Ω	L PH	$P_{OUT,s}$ W	η_{calc} %	η_{simul} %
0	0	14.07	2.31	470	1.03	100	99.6
0.08	0.02	15.30	2.06	432	1.03	95.0	94.8
0.16	0.04	16.91	1.80	390	1.03	89.2	88.7
0.24	0.06	19.16	1.51	342	1.04	82.5	81.9
0.32	0.08	22.88	1.17	285	1.04	73.6	72.8
0.40	0.10	36.34	0.59	175	1.05	54.0	52.0

V. Conclusions

A new formulation of analysis of Class E amplifier losses has been presented in this paper. The formulation is suitable for practical design, since the procedure starts with the design specifications P_{OUT} and V_{DD} and device characteristics R_{ON}. Based on these parameters the amplifier efficiency and optimum component values are solved. Additionally, a new upper bound for the switch losses has been found. R_{ON} is required to be less than approximately $0.1(V_{DD})^2/P_{OUT}$ in order to get the amplifier to operate in the Class E mode. The analysis provides better initial component values for practical Class E design than has been available previously. The validation simulations show excellent match with the calculated values.

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